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Lie Group Analysis of MHD Free Convective, Dissipative Boundary Layer Flow past a Porous Vertical Surface Under The Influence Of Thermal Radiation, Chemical Reaction And Constant Suction

K.V.Chandra Sekhar *

Abstract

The present paper describes MHD free convective, dissipative boundary layer flow past a vertical porous surface under the influence of thermal radiation, chemical reaction and constant suction, with the effect of uniform magnetic field which is applied normal to the surface is studied. The governing equations are solved using scaling group of transformations. The system remains invariant due to relations between the parameters of the transformations. By applying these transformations momentum equation, energy equation and diffusion equation reduced to non – linear 3rd and 2nd order ordinary differential equations. These equations are solved numerically. Finally the effects of various physical parameters of the flow are analyzed graphically using MATLAB.

Keywords:

Boundary layer; Free convection; Chemical reaction; MHD; Radiation; Porous medium; Vertical surface.

Author correspondence:

Department of Mathematics
K l university, Vaddeswaram – 522502, India

1. Introduction

There exist flows which are caused not only by the temperature differences but also by concentration differences as a result of which rate of heat transfer takes place. In many transport processes in industry the above said phenomena takes place. This phenomenon frequently exists in food processing and polymer production ([1]).

NOMENCLATURE

C^*	Concentration	T	Temperature
C_∞	Fluid concentration far away from the wall	u^* v^*	velocity components
c_p	specific heat at a constant pressure	u	Non dimensional velocity
D	Mass diffusivity	v_0	Suction velocity
E	Eckert number	κ	Thermal conductivity
F	Radiation parameter	ν	Kinematic viscosity
Gm	Mass Grashof number	σ	Electrical conductivity

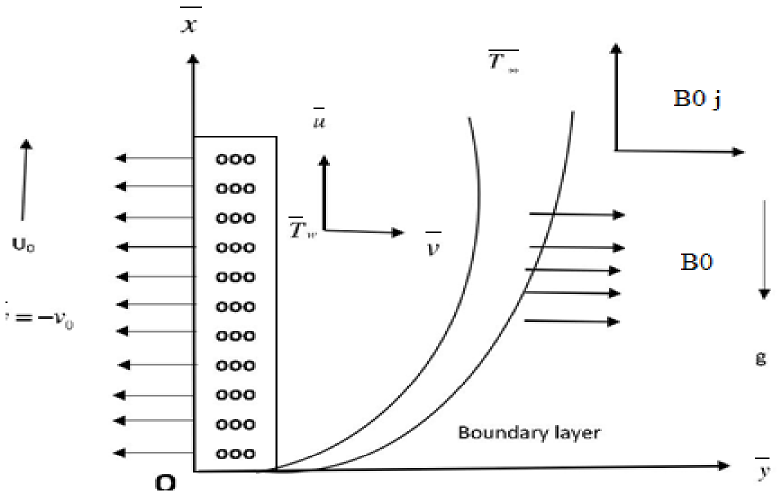
* Doctorate Program, Linguistics Program Studies, Udayana University Denpasar, Bali-Indonesia (9 pt)

Gr	Thermal Grashof number	μ	dynamic viscosity
g	gravitation due to acceleration	β_T	Coefficient of volume expansion
k_p	Permeability of porous medium	τ	skin friction non - dimensional
$k_{\lambda w}$	Absorption coefficient	θ	Non dimensional temperature
M	Magnetic parameter	W	Wall
Nu	Nusselt number	∞	Far away from the wall
$\Pr$	Prandtl number		Prime denotes differentiation with respect to y
q_r	Radiative flux		
Sc	Schmidt number		
T_∞	Fluid temperature for away from the wall		

Free convective flows have number of applications in industries like geothermal systems, fiber and granular insulation. There are so many applications of convective flow through porous medium in geothermal energy recovery, thermal energy storage, oil extraction, and flow through filtering devices. Raju and Varma [2], Magyari et al. [3], Ravikumar et al. [4], Chamkha [5], Makinde and Mhone [6], Hayat et al. [7], etc. are the few to make a mention who contributed in this area. Engineering devices, at high temperatures such as, gas, can be ionized and becomes a good electrical conductor. Soundalgekar and Takhar [8], studied the effect of radiation on the natural convection flow of a gas past a semi-infinite plate using the Cogley–Vincentine–Gilles equilibrium model. For the same gas Takhar et al. [9] included the effect of radiation on the MHD free convection flow past a semi-infinite vertical plate. Later, Hossain et al. [10] studied the effect of radiation on free convection from a porous vertical plate. Muthucumarswamy and Kumar [11] studied the thermal radiation effects on moving infinite vertical plate in the presence of variable temperature. Mazumdar and Deka [12] studied MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation. Radiation and mass transfer effects on a free convection flow through a porous medium bounded by a vertical surface were considered by Raju et al. [13]. Satyanarayana et al. [18] studied the effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system. The presence of a foreign mass in a fluid causes some kind of chemical reaction. In several industrial applications, such as, polymer production, manufacturing of ceramics or glassware and food processing, a chemical reaction undergoes between a foreign mass and the fluid in which the plate is moving. Muthucumarswamy [14] studied the influence of a chemical reaction on a moving isothermal vertical surface with suction. Recently, Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion was invesrtigated by Manivannan et al. [15] Effect of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source was investigated by Sharma et al. [16]. Mahapatra et al. [17] studied the effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Raju et al.[20] studied heat and mass transfer effects on a steady flow of viscous fluid through a porous medium bounded by a porous surface subjected to suction with a constant viscosity in the presence of radiation and homogenous chemical reaction of first order, which is an extension to the work of Mahapatra et al. [17]. Here an attempt was done to study the above said problem by using Lie group method. The graphical analysis is in good agreement with that of Raju et al[20].

2. Mathematical model

A viscous, incompressible, electrically conducting and radiating fluid through a porous medium was considered occupying a semi-infinite region of the space bounded by a vertical infinite surface. The x* axis was taken along the surface in an upward direction and the y* axis was normal to it. A uniform magnetic field B_0 is assumed to be applied normal to the surface. The properties of a fluid are assumed to be constant except for the density in the body force term.



Geometry of the problem

In addition a chemically reactive species is assumed to be emitted from the vertical surface into a hydrodynamic flow field. It diffuses into the fluid, where it under goes a homogenous chemical reaction. The reaction is assumed to take place entirely in the stream. Then the fully developed flow under the above assumptions through a highly porous medium is described by the following equations:

$$\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$v^*\frac{\partial u^*}{\partial y^*}=\nu\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta_T(T^*-T^*_\infty)+g\beta_c(C^*-C_\infty)-\frac{\sigma B_0^{\,2}}{\rho}u^*-\frac{\nu u^*}{k_p} \tag{2}$$

$$v^*\frac{\partial T^*}{\partial y^*}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\nu}{c_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y^*}g\beta_c(C^*-C_\infty) \tag{3}$$

$$v\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}-k_c(C-C^*_\infty) \tag{4}$$

The corresponding boundary conditions are

$$u^*=0,T^*=T_w,C^*=C_w \text{ at } y=0$$
$$u^*\to 0,T^*\to T_\infty,C^*\to C_\infty \text{ as } y\to\infty \tag{5}$$

Eqn (1) gives $v^*=-v_0$ (constant)

In the optically thick limit, the fluid does not absorb its own emitted radiation in which there is no self-absorption, but it does absorb radiation emitted by the boundaries. Cogley et al. [19] showed that in the optically thick limit for a non-gray gas near equilibrium as given below.

$$\frac{\partial q_r}{\partial y^*}=4(T^*-T^*_\infty)\int_0^\infty k_{\lambda w}\left(\frac{de_{b\lambda}}{dT^*}\right)_w d\lambda=4I_1\left(T^*-T_\infty\right) \tag{6}$$

Introducing the following non-dimensional quantities

$$\bar{u} = \frac{u*}{v_0}, \bar{y} = \frac{v_0 y^*}{v}, \theta = \frac{T - T_\infty^*}{T_w - T_\infty^*}, C = \frac{C - C_\infty^*}{C_w - C_\infty^*}, S_c = \frac{v}{D},$$

$$p_r = \frac{\rho\, c_p v}{k} = \frac{v}{\alpha}, M = \frac{\sigma B_0^2 v}{\rho v_0^2}, G_r = \frac{v g\, \beta_T (T_w - T_\infty)}{v_0^3} \quad (7)$$

$$G_c = \frac{v g\, \beta_c (C_w - C_\infty)}{v_0^3}, E = \frac{v_0^2}{c_p (T - T_\infty)}, k = \frac{v_0^2 k_p}{v^2}, (9)$$

$$k_0 = \frac{v k_c}{v_0^2}, F = \frac{4 I_1 v}{\kappa v_0}$$

Substituting (7) into equations (1) - (4) and dropping the bars, we obtain

$$v^* = -v_0 \quad (8)$$

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = -Gr\theta - GmC - \left(M + \frac{1}{k}\right)u \quad (9)$$

$$\frac{\partial^2 \theta}{\partial y^2} + \Pr \frac{\partial \theta}{\partial y} = -Ec \Pr \left(\frac{\partial u}{\partial y}\right)^2 + F\theta \quad (10)$$

$$Sc \frac{\partial C}{\partial y} = v \frac{\partial^2 C}{\partial y^2} - KcScC \quad (11)$$

The corresponding boundary conditions take the form

$$u = 0,\ \theta = 1,\ C = 1 \text{ at } y = 0 \quad (12)$$

$$u \to 0,\ \theta \to 0,\ C \to 0 \text{ as } y \to \infty \quad (13)$$

3. Solution of the Problem

In order to solve the coupled nonlinear system of Eqs. (8) – (11) with the boundary conditions (12 -13), the following similarity transformations are used.

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations

$$x^* = xe^{\varepsilon\alpha_1},\ y^* = ye^{\varepsilon\alpha_1}, \psi^* = \psi e^{\varepsilon\alpha_3},\ u^* = ue^{\varepsilon\alpha_3},$$

$$v^* = ve^{\varepsilon\alpha_5},\ \theta^* = \theta e^{\varepsilon\alpha_6},\ C^* = Ce^{\varepsilon\alpha_7} \quad (14)$$

By using the stream function $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ and substituting into equations (9 - 11) we get

$$e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\psi}{\partial y^3}+e^{\varepsilon(2\alpha_2-\alpha_3)}\frac{\partial^2\psi}{\partial y^2}=-Gre^{-\varepsilon\alpha_6}\theta-Gme^{-\varepsilon\alpha_7}\varphi+\left(M+\frac{1}{k}\right)e^{\varepsilon(\alpha_2-\alpha_3)}\frac{\partial\psi}{\partial y} \tag{15}$$

$$e^{\varepsilon(2\alpha_2-\alpha_6)}\frac{\partial^2\theta}{\partial y^2}+\Pr e^{\varepsilon(\alpha_2-\alpha_6)}\frac{\partial\theta}{\partial y}=-E\Pr\left(e^{\varepsilon(2\alpha_2-\alpha_3)}\frac{\partial\psi}{\partial y}\right)^2+Fe^{-\varepsilon\alpha_6}\theta \tag{16}$$

$$e^{\varepsilon(2\alpha_2-\alpha_7)}\frac{\partial^2 C}{\partial y^2}+Sce^{\varepsilon(\alpha_2-\alpha_7)}\frac{\partial\theta}{\partial y}-k_0ScC=0 \tag{17}$$

The above equations are invariant only if

$$\begin{aligned}&3\alpha_2-\alpha_3=2\alpha_2-\alpha_3=-\alpha_6=-\alpha_7=\alpha_2-\alpha_3\\&2\alpha_2-\alpha_6=\alpha_2-\alpha_6=-\alpha_6=4\alpha_2-2\alpha_3\\&2\alpha_2-\alpha_7=\alpha_2-\alpha_7,\alpha_7=0\end{aligned} \tag{18}$$

These relations gives $\alpha_2=-\alpha_1,\ \alpha_2=\alpha_3=\alpha_4=\alpha_6=\alpha_7=0,\alpha_5=-\alpha_1$ (19)

Thus the set of transformations Γ reduces to one parameter group of transformations
As follows

$$x^*=xe^{\varepsilon\alpha_1},\ y^*=y,\psi^*=\psi,u^*=u,v^*=ve^{-\alpha_1},\theta^*=\theta,C^*=C \tag{20}$$

The auxiliary equations are

$$\frac{dx}{x\alpha_1}=\frac{dy}{0}=\frac{d\psi}{0}=\frac{du}{0}=\frac{dv}{-v\alpha_1}=\frac{d\theta}{0}=\frac{dC}{0} \tag{21}$$

Solving the above equations we get the transformations

$$y=\eta,\psi=f(\eta),\theta=\theta(\eta),C=C(\eta) \tag{22}$$

Substituting these in above equations (7) we obtain the following system of
non – linear equations

$$f'''(\eta)+f''(\eta)=-Gr\theta(\eta)-GmC(\eta)-(M+1/k)f'(\eta) \tag{23}$$

$$\theta''(\eta)+\Pr\theta'(\eta)=-Ec\Pr f''(\eta)^2+F\theta(\eta) \tag{24}$$

$$C''(\eta)+ScC'(\eta)-k_0ScC=0 \tag{25}$$

The corresponding boundary conditions takes the form

$$f'(\eta)=0,\theta(\eta)=1,C(\eta)=1 \text{ at } \eta=0 \tag{26}$$

$$f'(\eta)\to 0,\theta(\eta)\to 0,C(\eta)\to 0 \text{ as } \eta\to\infty \tag{27}$$

4. Results and Discussions

The influence of various parameters is discussed. Fig 1to 8 depicts the effect of various parameters on velocity profiles. Fig – 1, 2, 3 4and 8 shows the influence of Schmidt number, Prandtl number, Magnetic parameter, Radiation Parameter and non dimensional chemical reaction parameter on velocity profiles. It has been observed that increase of these leads to decrease of velocity. Moreover we can observe for individual values the velocity increase rapidly and falls down moving away from the plate.

Fig – 5, 6, 7 shows that with increase of mass Grashof number, thermal gashof number, permeability parameter velocity increases. With the increase of above parameters there is rapid increase of velocity profiles, but far away from the plate the effect is not significant.
Fig - 9 ,10 shows effect of prandtl number and Radiation parameter on temperature profiles. It has been observed that increase of these parameters results in decrease of temperature. Near the wall the temperature is high , but away from the plate effect is not significant.

Fig – 11 shows with increase of Ekert number temperature decreases.
Fig – 12, 13 shows near the plate increase of mass grashof number , thermal grashof number leads to increase of skin friction, but within short distrance from the plate it was observed that a reverse trend was observed.
Fig – 14 depicts concentration profiles under the influence of Magnetic parameter. It is noticed that increase of Magnetic parameter results in decrease of concentration. For individual values of Magnetic parameter concentration takes higher values nearby plate and decreases rapidly far away from the plate.

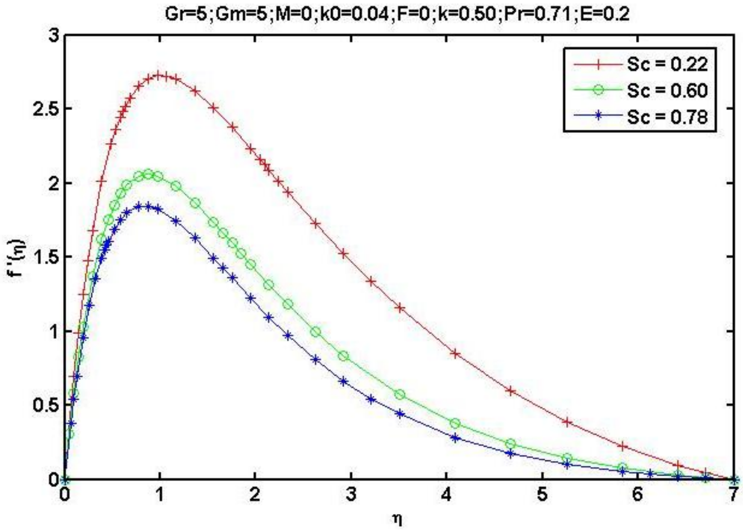


Fig – 1 : Effect of Schmidt number on velocity profiles

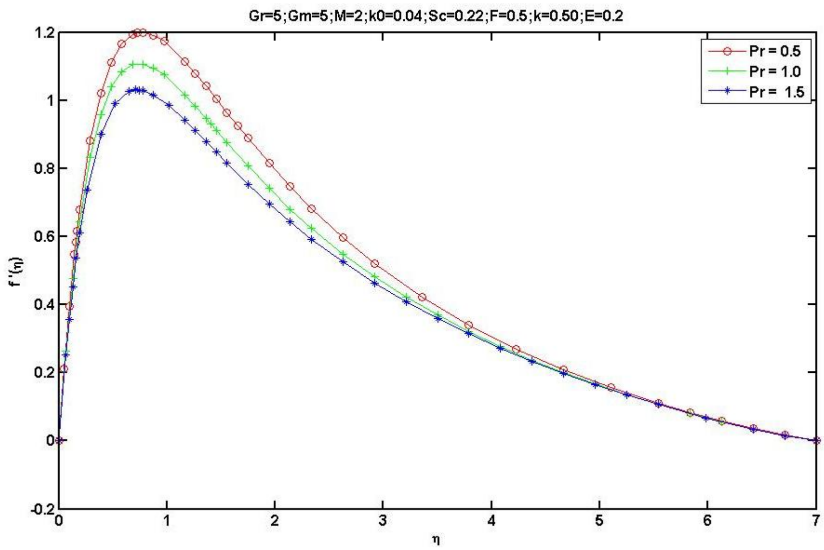


Fig – 2 : Effect of Prandtl number number on velocity profiles

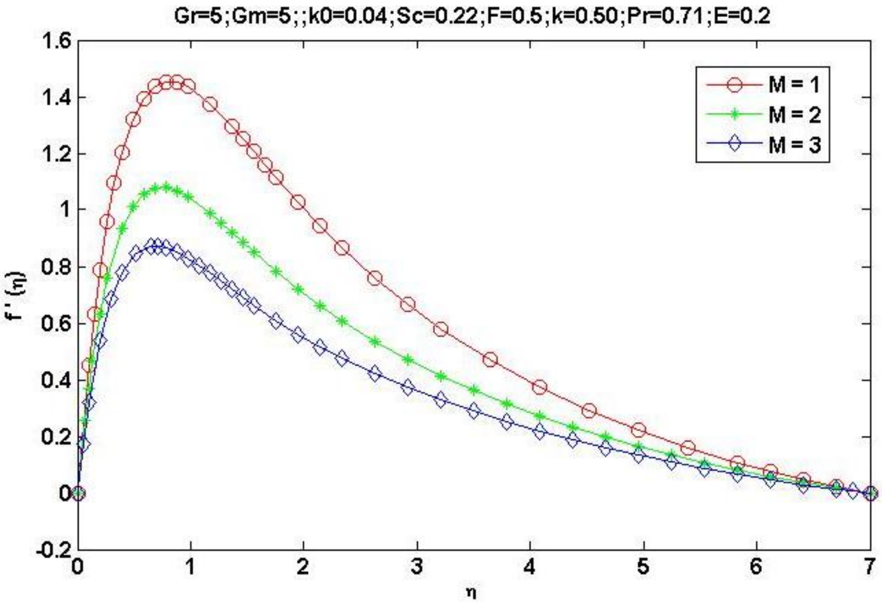


Fig – 3 : Effect of Magnetic parameter on velocity profiles

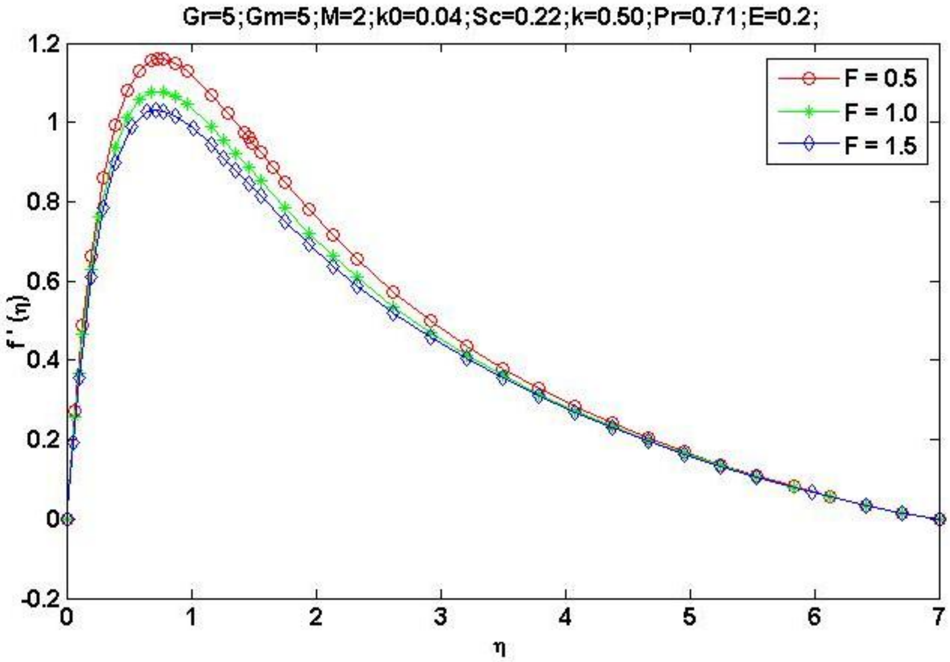


Fig – 4 : Effect of Radiation parameter on velocity profiles

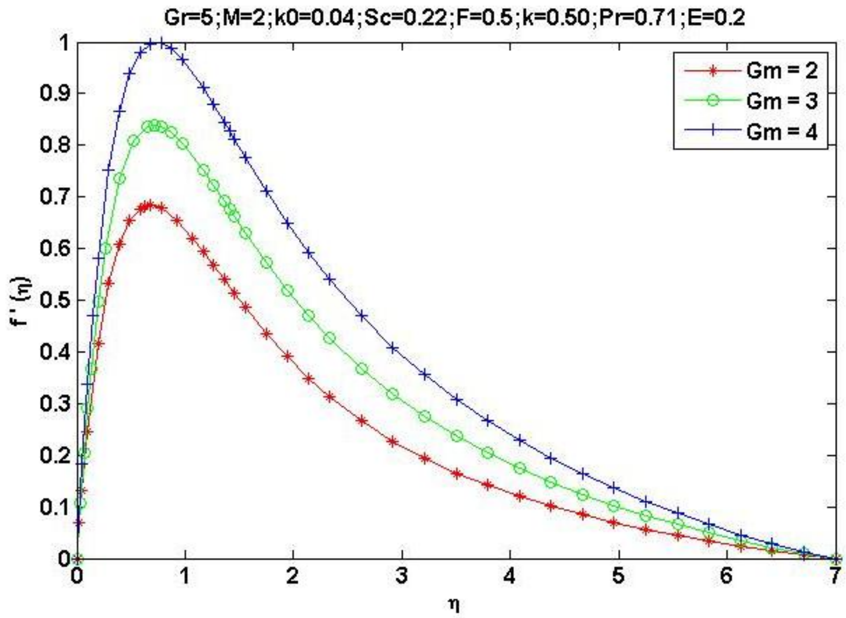


Fig – 5 : Effect of mass grashof number on velocity profiles

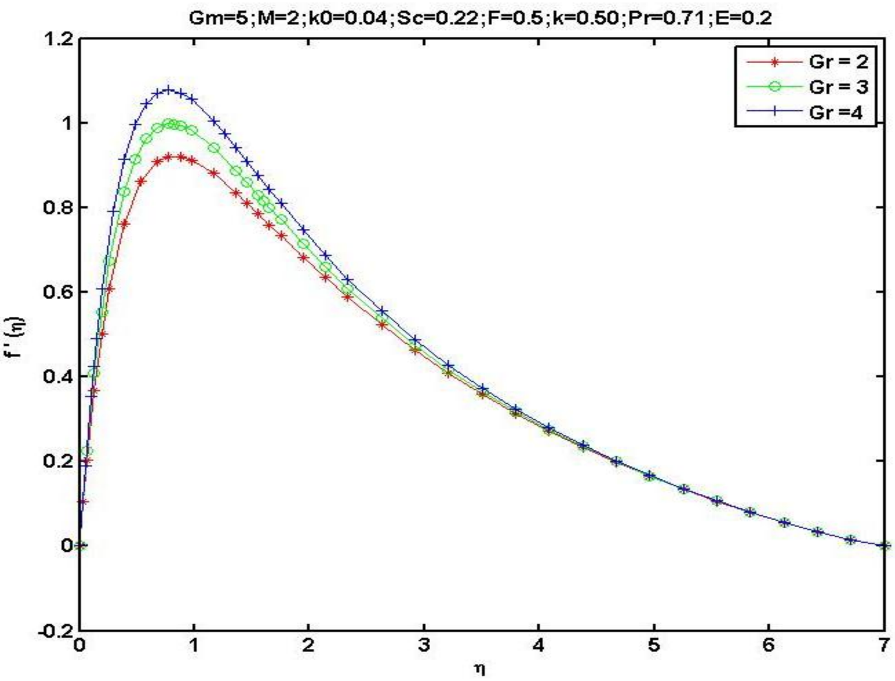


Fig – 6 : Effect of Thermal Grashof number on velocity profiles

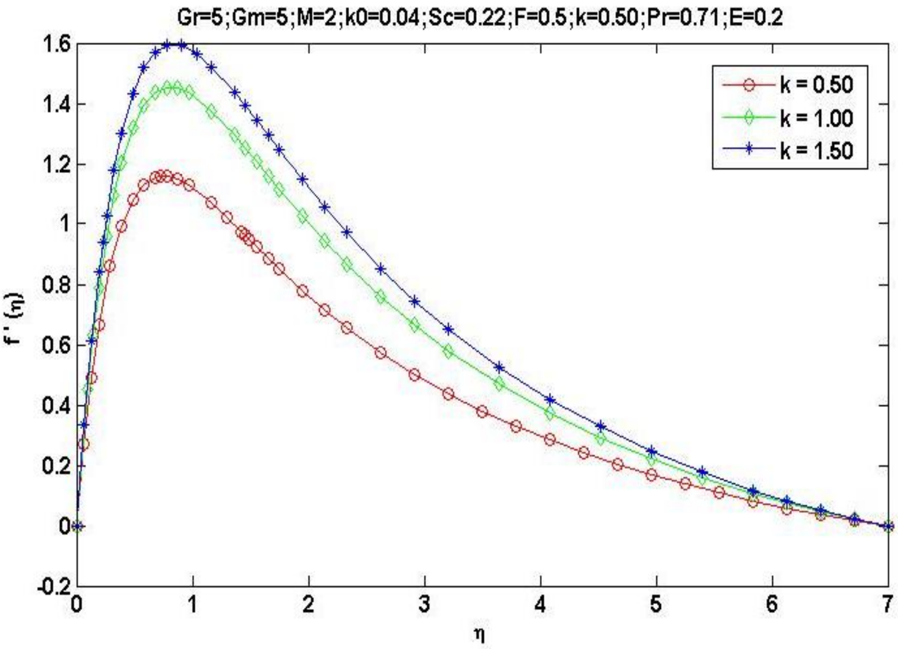


Fig – 7 : Effect of Permeability parameter on velocity profiles

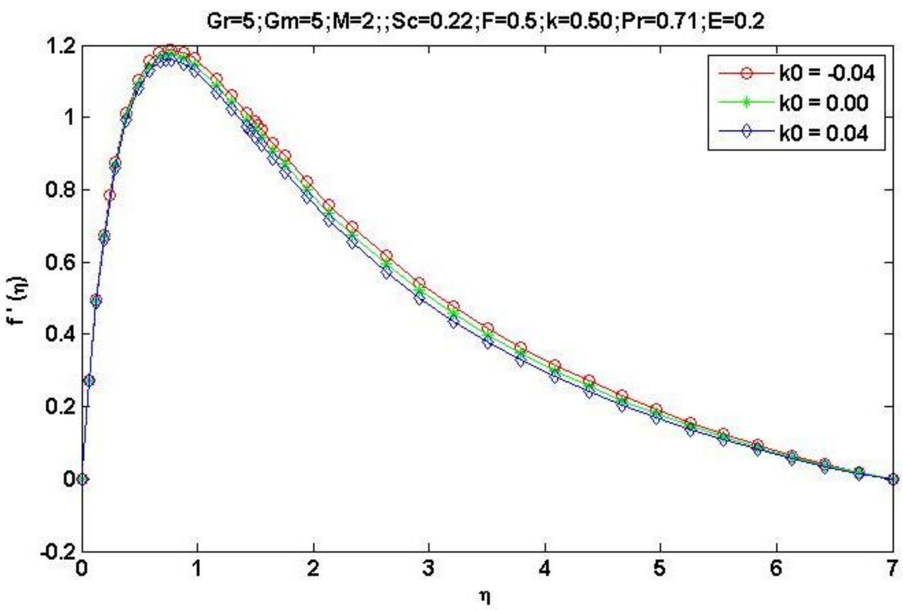


Fig – 8 : Effect of Non dimensional chemical reaction parameter on velocity profiles

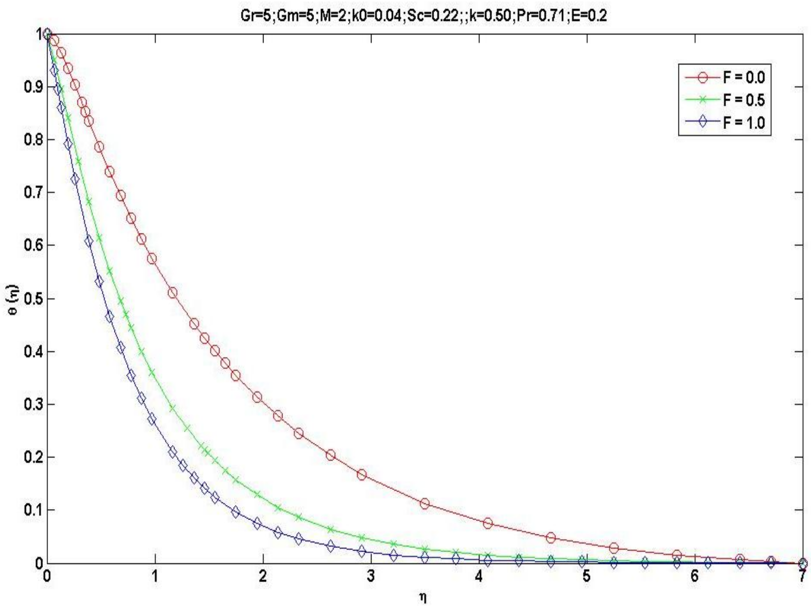


Fig – 9 : Effect of Radiation parameter on temperature profiles

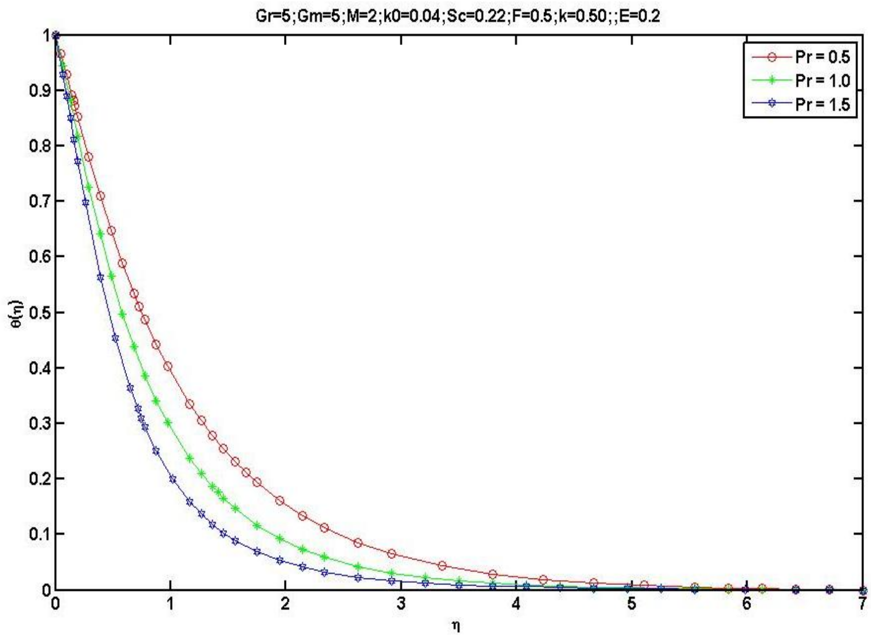


Fig – 10 : Effect of Prandtl number on temperature profiles

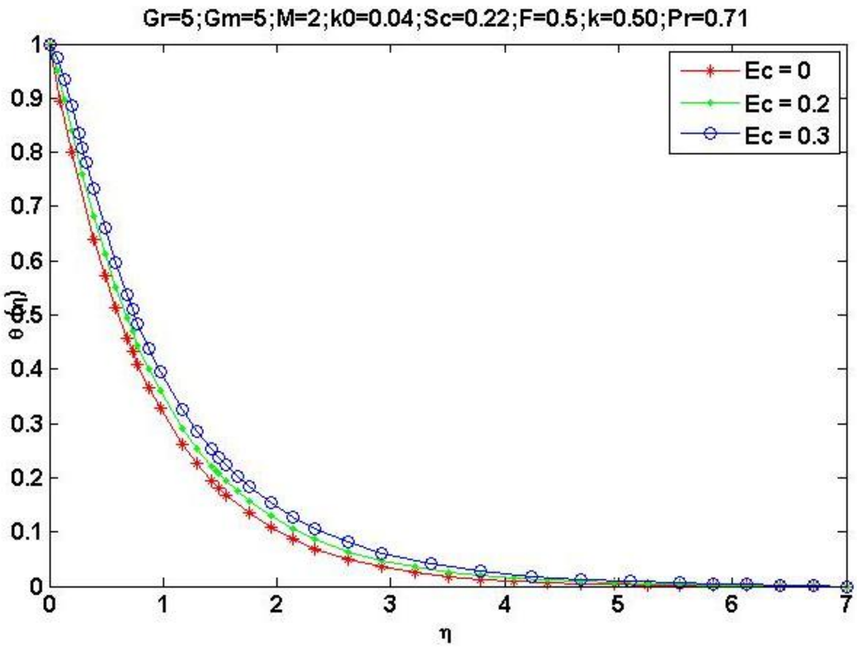


Fig – 11 : Effect of Ekert number on temperature profiles

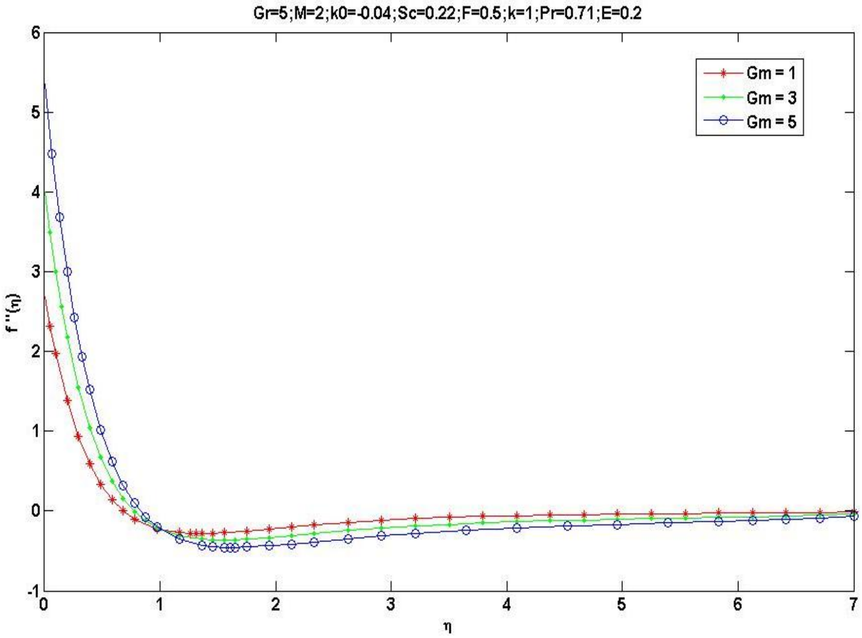


Fig – 12 : Effect of mass grashof number on Skin friction

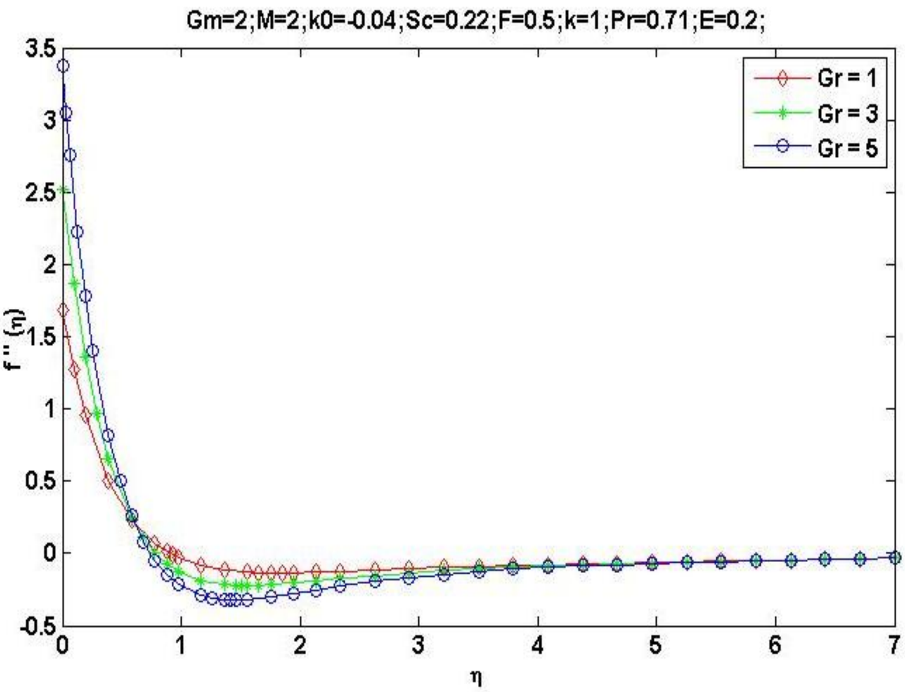


Fig – 13: Effect of Thermal grashof number on Skin friction

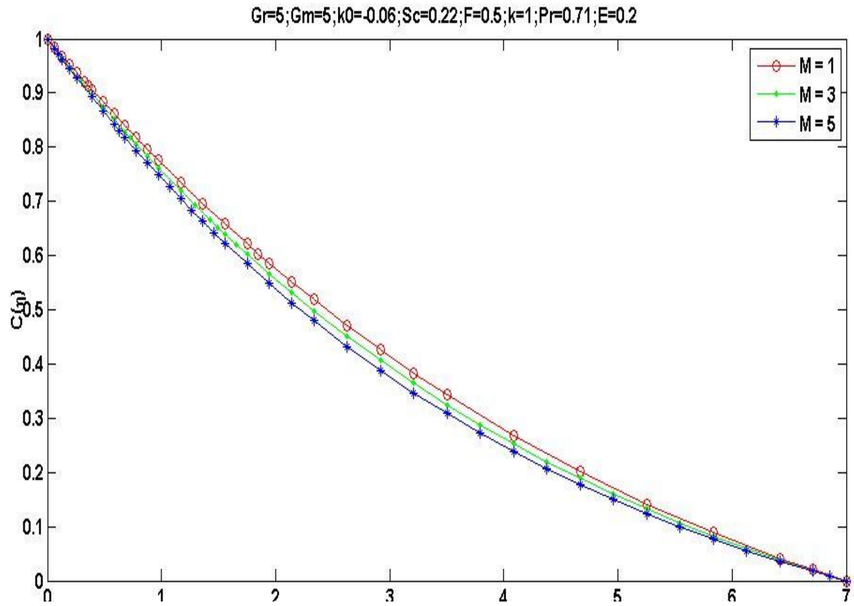


Fig – 14 : Effect of Magnetic parameter on Concentration profiles

5 Conclusions

1. Velocity decreases with increase of Schmidt number, Prandtl number, Magnetic parameter, Radiation parameter, non dimensional chemical reaction parameter.
2. Velocity increases with increase of mass Grashof number , thermal Grashof number, permeability parameter.
3. Temperature decreases with increase of radiation parameter , Prandtl number, whereas decreases with increase of Ekert number.
4. Increase of magnetic parameter leads to decrease in concentration profiles. For individual values of magnetic parameter there is exponential decrease of concentration of the spices.

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